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<u>Narrowband Frequency Modulation:</u>

Consider equation above,

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t)) =$$
$$A_c \cos 2\pi f_c t.\cos(\beta \sin(2\pi f_m t)) - A_c \sin 2\pi f_c t.\sin(\beta \sin 2\pi f_m t)$$

βIs small compared to one radian, we may approximate

$$\cos(\beta \sin 2\pi f_m t)) \approx 1$$

and

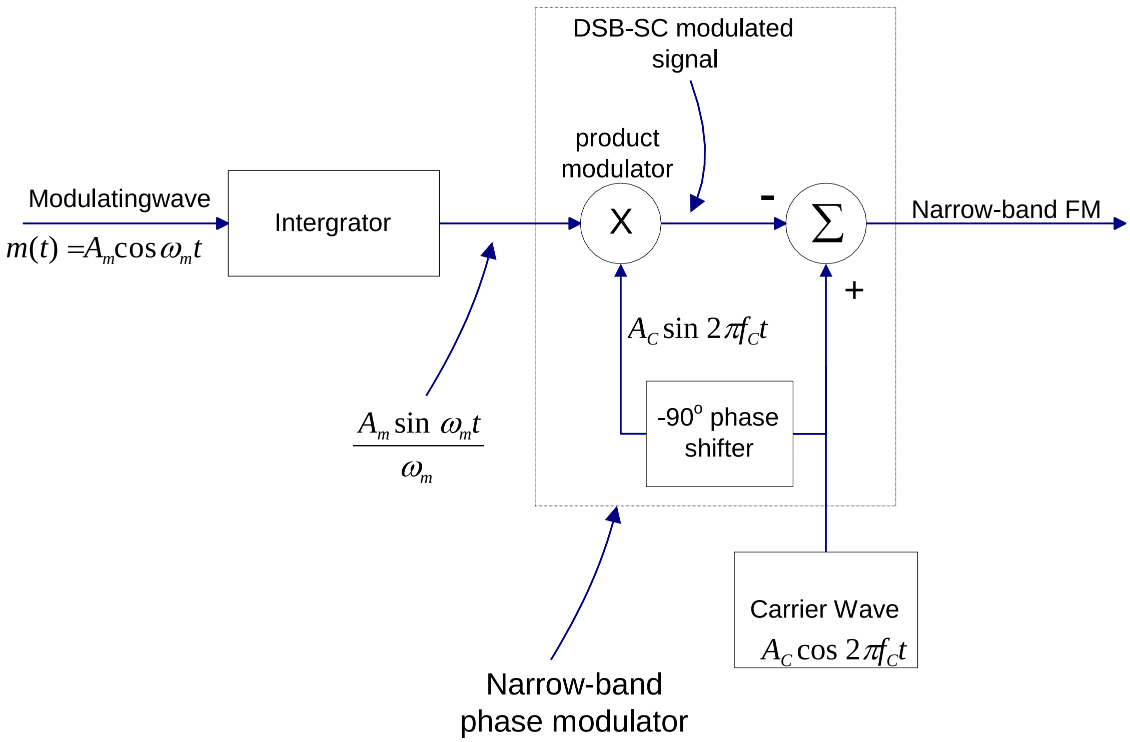
$$\sin(\beta \sin(2\pi f_m t) \approx \beta \sin(2\pi f_m t)$$

Hence equation2.18 becomes

$$s(t) \approx A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t).\sin(2\pi f_m t)$$

Narrowband FM signal

Equation 2.19 can be implemented as follows (Fig2.4).



Method for generating narrowband FM signal.

Ideally, an FM signal has a **constant envelope** and for the case of a sinusoidal modulating frequency f f_m, the angle $\theta_i(t)$ is also sinusoidal with the same frequency.

However the modulated signal produced by the narrowband modulator of Fig differs from this ideal condition in two fundamental respects:

- The envelope contains residue amplitude modulation and therefore varies with time.
- For a sinusoidal modulating the angle $\theta_i(t)$ Contains harmonic distortion in the form of third and higher-order harmonic of the modulation frequency f_m.

However, if β(modulation index) is restricted to $\beta \leq 0.3$ radians, the effect of residual AM and harmonic are limited to negligible levels.

Returning to equation 2.19(pp.127),

$$s(t) = A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t).\sin(2\pi f_m)t$$

Narrowband FM signal

$$s(t) = A_c \cos 2\pi f_c t + \frac{1}{2}\beta A_c \left[\cos 2\pi(f_c + f_m)t - \cos 2\pi(f_c - f_m)t\right]$$

Narrowband FM signal

Consider an AM signal equation

$$s(t) = A_c \cos(2\pi f_c t) + \frac{MA_c}{2}\left[\cos 2\pi(f_c + f_m)t + \cos 2\pi(f_c - f_m)t\right]$$

μ– Modulation factor of AM

AM signal

Comparing equation, we see that in the case of sinusoidal modulation, the Basic difference between an AM signal $ a narrowband FM signal is that the algebraic sign of the lower side frequency in the narrowband FM is reversed.
Thus, a narrowband FM signal requires essentially the same transmission band width (i.e.2 f_m) as the AM signal.

Wideband Frequency Modulation:

The FM signal itself is given by

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t)$$

We wish to determine the spectrum of the single tone FM signal above, for an arbitrary value of the modulation index β.

Assuming $\beta > 1$ (wideband frequency modulation we may write the above FM signal as

$$s(t) = A_c[\cos w_c t . \cos\beta \sin w_m t) - \sin w_c t . \sin(\beta \sin w_m t)]$$

- S(t) is no periodic , f_c is an integral multiple of f_m.
- We assume that f_c is large enough compared to the bandwidth of the FM signal.

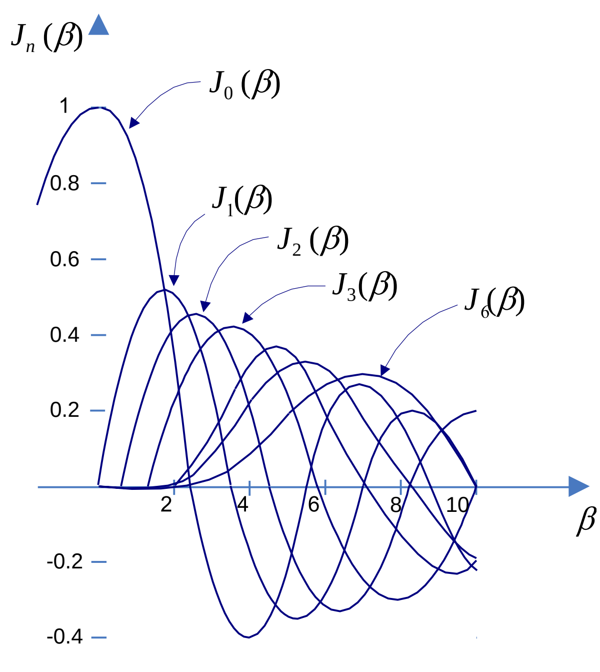
We know that:

$$\cos(\beta \sin\omega_m t) = J_0(\beta) + \sum_{n\,even}^{\infty} 2J_n(\beta)\cos n\omega_m t$$

$$\text{Sin } (\beta \sin\omega_m t) = \sum_{n\,odd}^{\infty} 2J_n(\beta)\sin n\omega_m t$$

Where n is positive and $J_n(\beta)$ are coefficient of Bessel functions of the first kind, of order n argument β.

Figure below shows the Bessel function $J_n(\beta)$ versus modulation index β for different positive integer value of n.



Bessel function for n=0 to n=6

We can develop further insight into the behavior of the Bessel function $J_n(\beta)$,

(1) $J_n(\beta) = (-1)^n J_{-n}(\beta)$ For all n both positive & negative.

(2) For small values of the modulation index β, we have

$$J_0(\beta) = 1$$

$$J_1(\beta) = \frac{\beta}{2}$$

$$J_n(\beta) = 0 \qquad n > 2$$

(3) $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$

Substituting equations and expanding products of sines and cosines finally yields:

$$s(t) = A_c J_0 \beta \cos \omega_c t$$

$$+ \sum_{n\,odd}^{\infty} A_c J_n(\beta)\left[\cos(\omega_c + n\omega_m)t - \cos(\omega_c - n\omega_m)t\right]$$

$$+ \sum_{n\,even}^{\infty} A_c J_n(\beta)\left[\cos(\omega_c + n\omega_m)t + \cos(\omega_c - n\omega_m)t\right]$$

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t$$

↑

Wide-band FM

s(t) is the desired form for the Fourier series representation of the single tone FM signal s(t) for an arbitrary value of β.

The discrete spectrum of s(t) is obtained by taking the Fourier transform of both sides of equation 2.15 & we have:

$$s(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta(f - (f_c + nf_m)) + 5(f + (f_c + nf_m))\right]$$

↑

FM signal

From 2.15& 2.16 we may make the following observation:

(1) The spectrum of an FM signal contain a carrier component and an infinite set of side frequencies located symmetrically on either side of the carrier at frequency separations f f_m ,2 $f_{m,}$ $3f_m$,................................. (note in AM system a sinusoidal Modulating signal given rise to only one pair of side frequencies.

(2) For the special case of βsmall compared to unity ,only the Bessel coefficients $J_0(\beta)$ and $J_1(B)$ have significant values , so that the FM signal is effectively Composed of a carrier and a single pair of side frequency $f_c \pm f_m$. at

This situation corresponds to the special case of narrowband FM that was considered earlier.

$$s(t) = \frac{AC}{2} J_0(\beta)\left[\delta(f - f_c) + \delta(f + f_c)\right] + \frac{AC}{2} J_1(\beta)\left[\delta(f - (f_c + f_m)) + \delta(f + f_c + f_m))\right]$$

n=0 n=1

[Note: $J_0(\beta) = 1$, $J_1(\beta) = \frac{\beta}{2}$]